TOURING RACE CAR GEARBOX REDESIGN USING THE AGMA METHOD

Andrew Bowyer
Completed as part of undergraduate requirements to earn a bachelors of science in Mechanical Engineering from Florida International University

Acknowledgments: Dr. Kevin Boutsen, Advisor

INTRODUCTION

In this report, the gearbox of a Honda Civic Type R TCR is redesigned using the AGMA method to increase performance as it pertains to two specific design goals. The redesign process is heavily based on the steps described in Shigley's Mechanical Engineering Design by R. G. Budynas and J. K. Nisbett. Specifications on the car's existing power transmission and engine performance have been measured and serve as the baseline for the modifications outlined in this document. The underlying intention of conducting this redesign is to demonstrate the opportunity for numerical optimization on gearbox specifications. For the purpose of this project, cost-saving measures will not be considered although future work may include further analysis on this subject. The project outcome is a set of gearbox specifications that include gear geometry, material selection, as well an automated process that can be repurposed for other (but similar) applications.

PROJECT REQUIREMENTS

The purpose of this project is to redesign a gearbox for a touring car competing in the FIA World Touring Car Cup – namely, a Honda Civic Type R TCR. The redesigned gearbox should accomplish the following core goals:

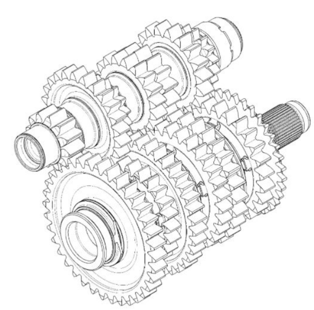
1. Increase the top speed of the car by at least 10%
2. Enhance vehicle acceleration from 0 to 120 mph

The core goals are the only two metrics by which success criteria may be determined but in addition to the core goals there are some constraints on the extent and scope of the redesign. These constraints include:

- The number of gears – six – should remain the same
- The final gear should not be changed
- Each gear mesh should be interchangeable and mounted between two parallel shafts
- Designing a gear shifter is out-of-scope
- The gearbox should fit within a window of 20 inches by 20 inches
- The engine performance should be considered as a constant
- Safety factor in bending is 1.5
- Safety factor in contact is 1.1

BASELINE GEARBOX OVERVIEW

Information on the existing gearing system and engine performance was provided to serve as a baseline for determining the degree to which the redesign achieves the core design goals.



The gear ratios for the existing gearbox are provided as follows:

Baseline Gearbox

Gear	Gear Ratio
First	13:35
Second	14:29
Third	17:27
Fourth	18:24
Fifth	18:21
Sixth	26:27
Final drive	15:57

Additionally, the torque and power curve of the engine are provided as follows:

Engine

RPM	Torque (ft•lbf)	Power (hp)
2500	141	67
3500	206	137
4500	259	222
5000	282	269
5500	293	307
6000	293	335
6500	288	357
7100	271	366

To determine land speed as it relates to the engine performance and gear ratios, the tire make and model are also provided. The tires are Yokohama N2800 250/660 R18 slicks.

REDESIGNED GEARBOX SUMMARY

Before describing the process by which the gearbox was designed, the final gearbox design will be briefly outlined. This should provide some background to facilitate understanding in the subsequent process description. Note that performance will not be included in this report until the process has been fully described.

Redesigned Gearbox

Gear	Gear Ratio
First	12:35
Second	12:29
Third	15:27
Fourth	16:24
Fifth	16:21
Sixth	21:18
Final drive	15:57

While the material properties of the existing gearbox were not specified (meaning modifications cannot be compared to baseline), determining materials for the redesigned gearbox was critical to the analysis. The redesigned gears are made of carburized and hardened, grade 2 steel.

The sum of face widths is 10.57 inches, this value indicates the theoretical minimum length of the gear system should it be desirable and feasible to remove all spacing between the gear meshes to produce the most compact gearbox possible. The individual face widths for each gear will be described in a later section.

PROCESS OVERVIEW

To achieve the gearbox specifications above, there were four major steps or phases conducted. The analysis was conducted using Python programming and three additional coding libraries: MatPlotLib (for displaying graphs), NumPy (for more involved mathematical operations outside

Python's native capabilities), and finally PrettyTable (to refactor console logs into easily consumable data tables). The four stages mentioned at the beginning of this section are adapted from the AGMA method as described in Shigley's Mechanical Engineering Design, chapters 13 and 14. The stages that make up the analysis procedure are as follows: Gearbox Specification, Basic Gear Geometry, Bending Stress Analysis, Contact Stress Analysis.

STAGE 1: GEARBOX SPECIFICATION

Gearbox specification is primarily focused on calculating the gear ratios and using these values to determine additional values based on constants for the gearbox, engine, and each mesh. In this stage, information is being generated about the existing and redesigned gears to inform later decisions and give feedback on those decisions.

First, the gear ratios for each of the six gears were calculated by dividing the number of teeth on the output gear by the number of teeth on the input gear. Each of these gear ratios are then multiplied by the final drive gear ratio to create six overall gear ratios.

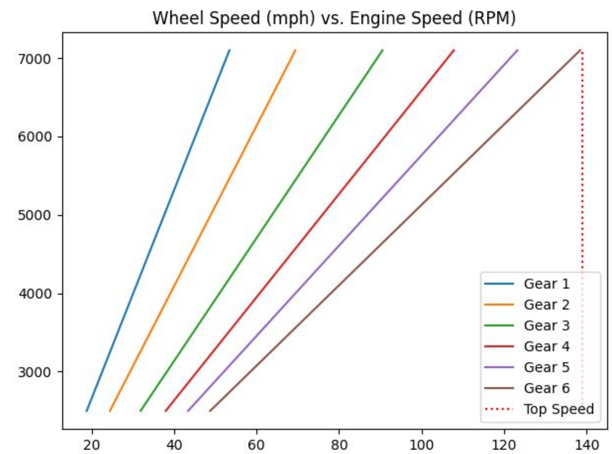
Next, the land speed of the vehicle can be determined by using the engine speed information, tire information, and gear ratios. The tire diameter was obtained from a [Yokohama product specification sheet](), and used to calculate the tire circumference. The following formula demonstrates how these values were used to calculate the land speed as a function of engine speed and gear ratio:

$$v(n_{ei}, m_i) = n_{ei}[rpm] * \frac{60\ minutes}{1\ hour} * C * \frac{1\ mile}{63360\ inch} * \frac{1}{m_i}$$

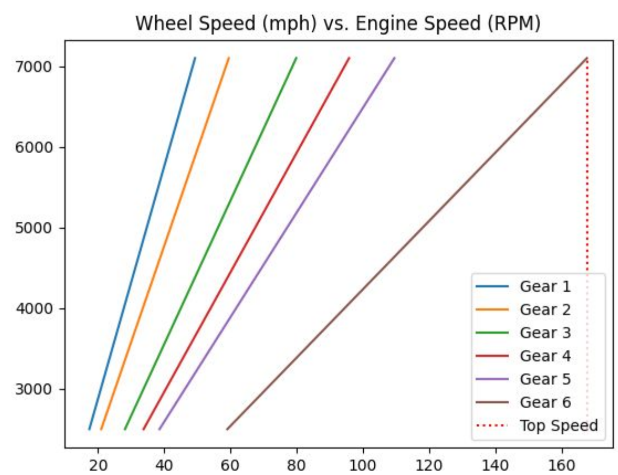
Where:

- v is the land speed, or wheel speed in mph
- m_i is the gear ratio for each gear
- n_{ei} is each of the tabulated engine speeds in RPM
- C is the tire circumference

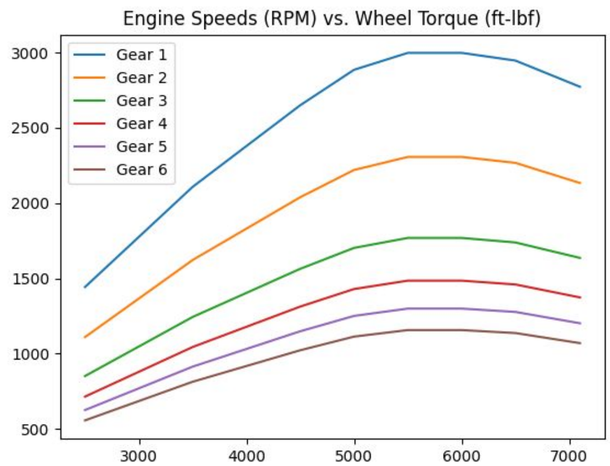
It's helpful to keep in mind that both gear ratio and engine speed have multiple values so that outcome of this operation is an array of values showing the vehicle speed if the engine is in any of the six gears at any of the eight tabulated engine speeds. The values for the baseline gearbox can be visualized in the following graph:



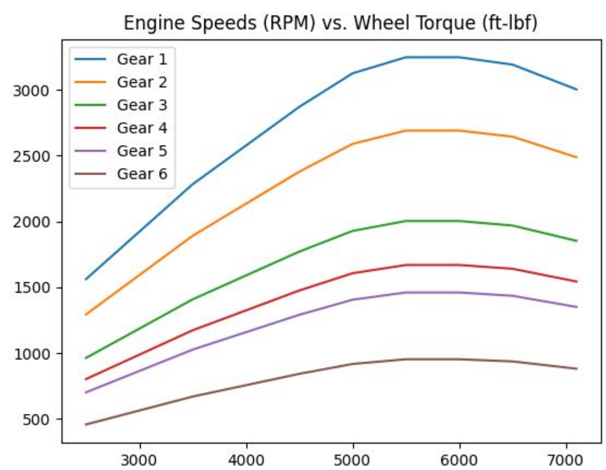
And for the redesign gearbox:



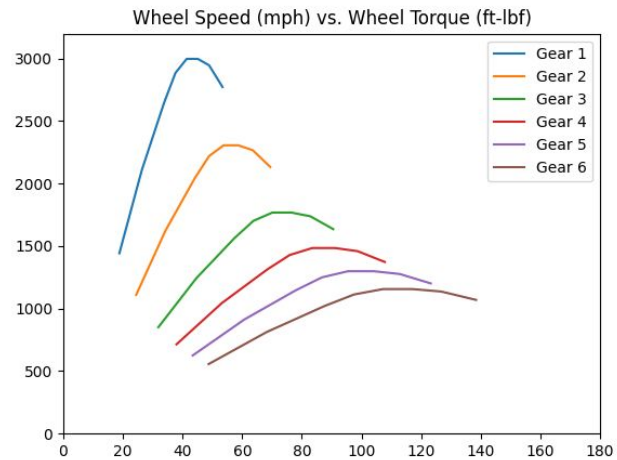
Next, the torque at the wheel was calculated by multiplying the gear ratio with the engine torque. Wheel torque in each gear at each tabulated engine speed is as follows:



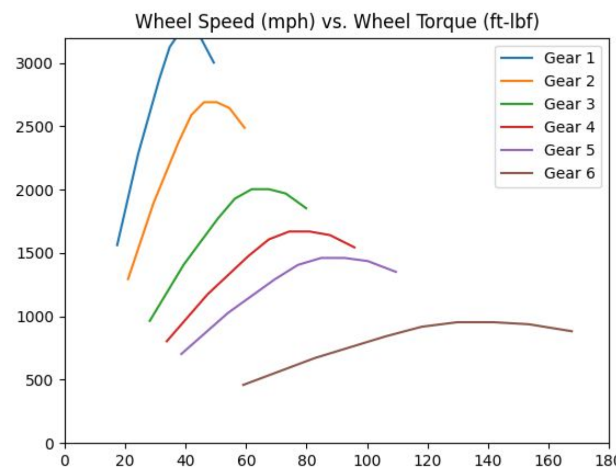
Again, for the redesigned gearbox:



While these graphs are useful in visualizing the array of data for each gearbox, they lack necessary insight into how to make performance-based design decisions. Specifically, these graphs do not provide enough information on how gear changes should be executed to drive the race car at near-peak capabilities. For this reason, the existing data was refactored into the following graphs that show land speed versus torque. For the baseline gearbox that looks like:



For the redesigned gearbox:



Analysis on these plots, preceding and proceeding plots will be included in the “results” portion of the report.

For purpose of procedure, these plots demonstrate the optimal torque vs. speed curve as the vehicle accelerates. Assuming that the vehicle operates on a clutch system so that speed and torque may start at 0 mph and 0 ft-lbf: this curve, can be envisioned as the continuous approximation starting at zero, continuing up through first gear, over the peak torque, down the tail end of gear 1, then following the tail ends of gears 2 through 6, and continuing until top speed is achieved.

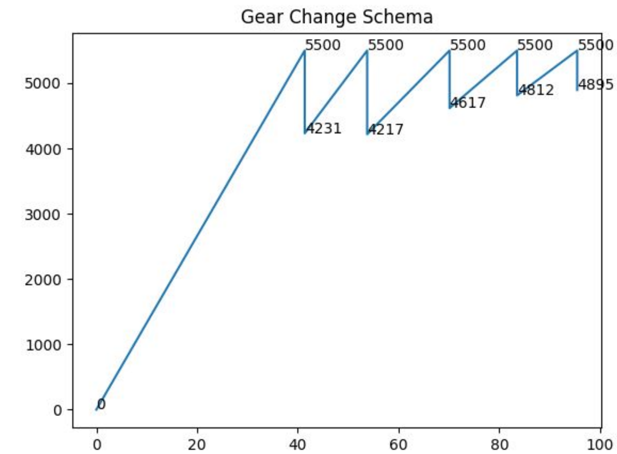
Because the max engine speed is 7100 RPM, this curve would never be approached in practical applications. For the purpose of modeling, max torque was used as the indicator for when gear change should occur. Additionally, to accommodate for the disparity between mathematical modeling (in which, max torque will have been instantaneously achieved) and real driving experience (in which, max torque must be for some time before the vehicle will actually experience the performance) the data point immediately following max torque was used for the calculations.

Using the information presented in the previous graphs, the optimal engine speeds for each of the gears can be calculated and plotted. The algorithm by which this is done is straightforward and direct.

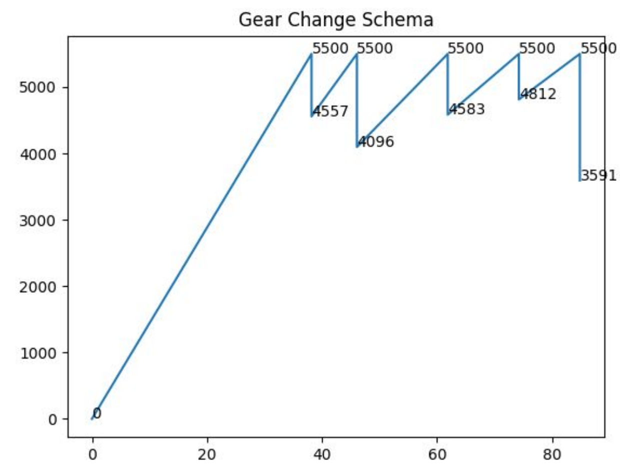
1. Use the "Wheel Speed vs. Wheel Torque" data to find the gear change speed in each gear
2. Use the "Wheel Speed vs. Engine Speed" data to find the engine speed at which gear change occurs for each gear.
3. Repeat the previous step, but record the engine speed for the current gear plus one. For example, if gear change in gear 1 occurs at 40 mph, then the engine speed for gear 2 at 40 mph should be recorded.

Using the information that can be collected using the described algorithm, two graphs were plotted as a reference for the optimal engine speeds in gear. It should be noted that each of these graphs only represent a virtual dataset and serve as visual aids for the back-end calculations of the program. The program itself does not rely on the graphs to make approximations of calculations.

The optimal engine speeds in gear for the baseline gearbox:



The optimal engine speeds in gear for the redesign gearbox:



Finally, for stage 1, the gear meshes were tested for interference. At this point a pressure angle, $\phi = 20°$, was selected as well as a value of $k = 1$ for full-depth teeth. The following formula from Shigley's Mechanical Design Engineering was used to determine the lowest value possible for the number of teeth on a pinion, N_p, with a predetermined gear ratio, $m = m_G$.

$$N_p = \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (1+2m)\sin^2\phi}\right)$$

If any interference was detected a console log was printed to indicate that modifications should be made. To summarize stage 1 the following tables were generated.

Baseline					
Gear	N_1	N_2	Overall Gear Ratio, m	Top Speed in Gear (mph)	Max Torque (ft-lbf)
1	13	35	10.23	53.4	2997.62
2	14	29	7.87	69.41	2306.33
3	17	27	6.04	90.53	1768.34
4	18	24	5.07	107.83	1484.53
5	18	21	4.43	123.24	1298.97
6	23	27	3.95	138.45	1156.22

Redesign					
Gear	N_1	N_2	Overall Gear Ratio, m	Top Speed in Gear (mph)	Max Torque (ft-lbf)
1	12	35	11.08	49.3	3247.42
2	12	29	9.18	59.49	2690.72
3	15	27	6.84	79.88	2004.12
4	16	24	5.7	95.85	1670.1
5	16	21	4.99	109.54	1461.34
6	21	18	3.26	167.74	954.34

STAGE 2: FUNDAMENTAL GEAR GEOMETRY

At this point, the baseline gearbox can be disregarded for the remainder of the procedure. As will be described in the "results" section, if we assume the following stages produce satisfactory results based on the stage 1 (meaning, no modifications are necessary), then it can be determined that the two core design goals have been achieved.

Moving forward through stage 2, 3, and 4, only the redesigned gearbox will be referenced.

The purpose of this stage is to determine whether a subset of the additional design constraints have been achieved. The design constraints considered are listed below.

- Each gear mesh should be interchangeable and mounted between two parallel shafts
- The gearbox should fit within a window of 20 inches by 20 inches

The design challenge posed by the first point is that each of the six gear meshes should have the same center-to-center distance if they are to be interchangeable and mounted on the same two shafts.

$$(r_P + r_G)_1 = (r_P + r_G)_2 = (r_P + r_G)_3 = (r_P + r_G)_4 = (r_P + r_G)_5 = (r_P + r_G)_6$$

To overcome this challenge, the following algorithm is executed by the program to consistently produce a set of six gear meshes with identical center-to-center distances.

1. Determine a minimum desired gear pitch
2. Calculate the gear and pinion diameter based on the minimum gear pitch
3. Determine the center-to-center distances by summing the radius of the gear and pinion for each mesh
4. Determine the shortest center-to-center distance of the six gear meshes
5. Increase the pitch of each mesh until they achieve the shortest center-to-center distance as well

It should be noted that this algorithm is under the assumption that each of the gears can be custom designed to achieve a specific gear pitch (regardless of whether it falls within a standard or not). The function takes a starting gear pitch for each mesh as an input and returns a pitch for each mesh as well a calculation for gear diameters, and the center-to-center distance. In this context of this project, this method is favorable to a typical trial-and-error method as it only requires one decision (initial gear pitch) to be made. Additionally, the computation is simplified because the fundamental requirement for gear design is that for any two gears to form a mesh they must have the same pitch. The relation between diametral pitch, P, the number of teeth on a gear, N, and diameter, d is the basis for the calculations in the preceding algorithm.

$$P = \frac{N}{d}$$

The challenge posed by the second constraint (concerning the 20 inch window), is that the diameter of the largest and smallest gear on either shaft should not exceed a certain value as described by the circular geometry. Using a generalized case in which two gears of radii, r_P , and, r_G , are placed within a square of side length, s, the following relation was found.

$$(\frac{\sqrt{2}+1}{2})(r_P + r_G) \leq s$$

As described by the project goals, $s = 20$. With this relation, a pass or fail value was assigned to each mesh to determine whether it would reasonably satisfy the constraint.

A summary of the information used to address both design constraints can be found in the following table.

Redesign							
Gear	P	N_p	d_P	N_G	d_G	C-C Distance	P/F
1	4.9	12	2.45	35	8.12	5.285	pass
2	4.06	12	2.96	29	7.62	5.285	pass
3	4.3	15	3.49	27	7.08	5.285	pass
4	4	16	4	24	6.57	5.285	pass
5	3.5	16	4.57	21	6	5.285	pass
6	3.87	12	4.65	21	5.92	5.285	pass

STAGE 3: BENDING STRESS ANALYSIS

Once the fundamental geometry of the redesigned gearbox was determined, the bending stress could be analyzed in each mesh to evaluate whether an appropriate safety factor of 1.5 was achieved.

The highest level description of this stage is as follows:

1. Determine the stress present in the pinion and gear of each mesh
2. Determine the max allowable stress
3. Adjust design parameters, F and P, until actual stress is less than allowable stress
4. Calculate safety factor

The principle equation for the analysis is the AGMA bending stress equation.

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

To begin the analysis, the various parameters were determined.

- The overload factor, $K_o = 1.25$ was determined by using tabulated resources from Shigley's to account for "light shock" on the gears from vibration or sudden gear changes.
- The size factor, $K_s = 1$, was used as no size-related effects can be determined without prior evidence. This follows guidance from the AGMA standard.
- A rim thickness factor of $K_B = 1$ was used under the assumption that each shaft could designed so that $m_B \geq 1.2$.
- The load-distribution factor, K_m, was calculated under the assumption that the gears would extra-precision manufactured (this determines the tabulated A, B, and C values), that the gearing is adjusted after assembly ($C_e = 0.8$) and that the teeth are crowned ($C_{mc} = 0.8$). Each of these assumptions were made to minimize the impact of load-distribution as this simplifies the intensity of computation and as these

simplifications are not explicitly denied in the project requirements.

- The velocity factor, K_v, was calculated under the assumption that the quality number is $Q_v = 11$, for high-quality.
- The integrated geometry factor, J, was manually entered for each mesh from the tabulated values in Shigley's. The console printed the number of teeth on the pinion and gear to aid in this manual process. The J values were 0.21, 0.21, 0.25, 0.27, 0.27, 0.31 for gears 1 through 6 respectively.

The load was calculated using the following relations, based on pitch-line velocity, power, and gear speed.

$$V = \frac{\pi d n}{12}$$

$$W^t = \frac{33000H}{V}$$

Finally, the face width was initially set to a median value by the following calculation. This value is later optimized.

$$F = \frac{4\pi}{P}$$

Using these values and the diametral pitches calculated in stage 2, the stress can be calculated in the pinion and gear for each mesh.

The allowable stress was then calculated using the following AGMA equation:

$$\sigma_{\text{all}} = \frac{S_t}{S_F}\frac{Y_N}{K_T K_R}$$

To calculate the allowable stress the various parameters were determined.

- The strength, $S_t = 65000\, psi$, was determined from a table. This value depends on the material selected, which in this case was carburized and hardened grade 2 steel. This material was initially selected as it had one of the high strengths in its class.
- The bending safety factor, $S_F = 1.5$, is given in the project requirements.
- The value for $Y_N = 1$ is made under the assumption that the gearbox will undergo the standard condition of 10^7 cycles.
- The reliability factor, $K_R = 1.25$, was determined by manually identifying the value for a reliability of $R = 0.999$. The assumption is that a touring car in a world-class race should have higher than standard reliability, as a gearbox failure could cause a huge setback
- The temperature factor, $K_T = 1$, is set in the assumption that the gearbox will not exceed a value of 250°

With both the allowable and actual stresses calculated, analysis to optimize the face width can be executed. For each mesh, the stress was recursively compared with the allowable stress and categorized into two conditions: either the stress was greater than the allowable stress – indicating a failure, or the stress was less than the allowable stress – indicating there was margin for improving the gearing within the constraints. For the meshes that met the first condition, the face width was incrementally increased until stress was less than allowable stress. For the second condition, the opposite was done. The face width was increased incrementally until the stress was within a small range of the allowable stress. In plain engineering english this process is relatively straightforward but the computation required more involved functions to recursively determine the stress values and conduct on-going status checks on the meshes to ensure successful completion of the design goal.

A summary of the pinion and gear bending stresses can be found below.

Redesign - Pinion

Gear	P (in^{-1})	F_i (in)	F (in)	σ (psi)	σ_{all} (psi)	S_F	$S_F \geq 1.5$
1	4.9	2.56	3.36	33383.39	34666.67	1.56	pass
2	4.06	3.1	2.3	30987.62	34666.67	1.68	pass
3	4.3	2.93	1.73	29868.93	34666.67	1.74	pass
4	4	3.14	1.34	28350.69	34666.67	1.83	pass
5	3.5	3.59	0.99	29593.05	34666.67	1.76	pass
6	3.87	3.25	0.85	28112.33	34666.67	1.85	pass

Redesign - Gear

Gear	P (in^{-1})	F_i (in)	F (in)	σ (psi)	σ_{all} (psi)	S_F	$S_F \geq 1.5$
1	4.9	2.56	3.36	10793.1	34666.67	4.82	pass
2	4.06	3.1	2.3	12718.99	34666.67	4.09	pass
3	4.3	2.93	1.73	15370.54	34666.67	3.38	pass
4	4	3.14	1.34	17781.93	34666.67	2.92	pass
5	3.5	3.59	0.99	22922.33	34666.67	2.27	pass
6	3.87	3.25	0.85	22440.31	34666.67	2.32	pass

STAGE 4: CONTACT STRESS ANALYSIS

The highest level description of this stage is as follows:

1. Determine the contact stress present in the pinion and gear of each mesh
2. Determine the max allowable contact stress
3. Evaluate whether material is satisfactory
4. Calculate safety factor

The principle equation for the analysis is the AGMA contact stress equation.

$$\sigma_C = C_p\sqrt{W^t K_o K_v K_s \frac{K_m}{d_P F}\frac{C_f}{I}}$$

Although many of the factors have already been discussed or calculated for bending stress, there are a few more factors that are exclusive to contact stress calculations.

The elastic coefficient, $C_p = 2300$, was determined as a tabulated value from steel-to-steel contact between the gear and pinion of each mesh.

The surface condition factor, $C_f = 1$, was set assuming there are no known surface defects

The geometry factor, I, is calculated using the following equation:

$$I = \frac{\cos\phi_t \sin\phi_t}{2m_N}\frac{m_G}{m_G+1}$$

Where $m_N = 1$ because the gearbox is using spur gears.

With all of this considered, contact stress can be calculated for each pinion and gear in each mesh.

Next, the allowable contact stress is calculated using the AGMA allowable contact stress equation.

$$\sigma_{c,\text{all}} = \frac{S_c}{S_H}\frac{Z_N C_H}{K_T K_R}$$

Once again, many of the values are carried over from previous determinations but the exclusive contact stress factors are described below.

- The contact strength value, $S_c = 225000$ psi is manually obtained from a table by using the properties of carburized and hardened grade 2 steel
- The stress cycle factor, $Z_N = 1$, is given under the same conditions of 10^7 cycles
- The contact safety factor, $S_H = 1.1$, is given in the project description
- The hardness ratio factor, $C_H = 1$, is set under the assumption that both the gear and pinion are the same material so that their brinell hardness values are equal. This satisfies the condition to simplify the C_H calculations

Because the design parameters involved in these calculations have already been predetermined in stage 3, this process is much less involved and any failures at this point would posit a return to previous design decisions. A summary of the contact stresses on the pinions and gears for each mesh can be found below.

Redesign - Pinion							
Gear	P (in^{-1})	F_i (in)	F (in)	σ_c (psi)	$\sigma_{c,all}$ (psi)	S_H	$S_H \geq 1.1$
1	4.9	2.56	3.36	149260.83	163636.36	1.21	pass
2	4.06	3.1	2.3	147554.61	163636.36	1.22	pass
3	4.3	2.93	1.73	148294.04	163636.36	1.21	pass
4	4	3.14	1.34	150478.62	163636.36	1.2	pass
5	3.5	3.59	0.99	158071.91	163636.36	1.14	pass
6	3.87	3.25	0.85	159794.7	163636.36	1.13	pass

Redesign - Gear							
Gear	P (in^{-1})	F_i (in)	F (in)	σ_c (psi)	$\sigma_{c,all}$ (psi)	S_H	$S_H \geq 1.1$
1	4.9	2.56	3.36	84869.91	163636.36	2.12	pass
2	4.06	3.1	2.3	94533.33	163636.36	1.9	pass
3	4.3	2.93	1.73	106379.63	163636.36	1.69	pass
4	4	3.14	1.34	119174.26	163636.36	1.51	pass
5	3.5	3.59	0.99	139119.88	163636.36	1.29	pass
6	3.87	3.25	0.85	142767.21	163636.36	1.26	pass

RESULTS

Considering the performance summary included at the end of this section, the core design goals can be evaluated for the gearbox redesign. Revisiting the core design goals:

1. Increase the top speed of the car by at least 10%
2. Enhance vehicle acceleration from 0 to 120 mph

First, the top speed of the baseline gearbox was calculated at approximately 138.5 mph. To increase this value by 10% the top speed of the redesigned gearbox should be at least 153 mph. The top speed achieved by the redesign is approximately 168 mph, indicating that the first design goal has been satisfied. This was done by changing the sixth gear from an underdrive setup (the pinion is driving, the gear is driven) to an overdrive setup (the gear is driving, the pinion is driven). With an overdrive mesh, the torque is decreased proportionally but this enables the gear speed to increase which relates to vehicle speed.

Second, the vehicle acceleration must be understood in mechanical terms. Vehicle acceleration is directly related to the torque at the wheels which is a function of the engine speed and the gear ratios. The torque in gears 1 through 5 was increased as compared to the baseline. The design decision that was made to elicit this result is to increase the gear ratio or make the gearing more "aggressive". This increased ratio has stress trade-offs but can be made to accommodate for the core design goal. Full torque analysis can be seen in the summary.

	Performance					
Gear	Gear Ratio		Top Speed in Gear (mph)		Max Torque (ft-lbf)	
	Baseline	Redesign	Baseline	Redesign	Baseline	Redesign
1	10.23	11.08	53.4	49.3	2997.62	3247.42
2	7.87	9.18	69.41	59.49	2306.33	2690.72
3	6.04	6.84	90.53	79.88	1768.34	2004.12
4	5.07	5.7	107.83	95.85	1484.53	1670.1
5	4.43	4.99	123.24	109.54	1298.97	1461.34
6	3.95	3.26	138.45	167.74	1156.22	954.34

FUTURE STUDY

While the core design goals and additional constraints were satisfied in this redesign there are two points that could be improved in future study to potentially improve performance and the engineering decision-making process for this project.

Firstly, in this implementation there are only two factors that are actively optimized through numerical methods. Those factors being the face width and the diametral pitch. While optimizing these two factors facilitated the remaining analysis in each stage, these values were optimized reactively. This means that their values were optimized with one or more decision decisions previously being made. Because of the highly numerical nature of the stress analysis and gear design, it would be possible to refactor the code to take a design goal and engine performance as an input then provide suggestions for values such as the gear ratios, pressure angle, brinell hardness or strength, and the other underlying values that are used to calculate the stress factors. In this arrangement, the code could be responsible for taking fundamental inputs, making suggestions, then making calculations based on the engineers “actual” values.

Secondly, in terms of implementation and execution there are some improvements that could be made to alleviate some computational intensity and improve user interaction. Namely, a more intuitive class structure for the code where there are four classes (gearbox, mesh, pinion, gear) with attributes associated with each of the necessary computation parameters as well as the calculated mechanical parameters. Introducing a GUI for user interfacing would additionally remove a lot of the metaphorical friction in scripting and decision-making.

RESOURCES

The code was written with the following dependencies: Python 3.7, MatPlotLib 3.3, and PrettyTable 0.7.2. The full 700+ lines of code can be viewed and cloned from the GitHub repository.